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Time-Averaged Gradient Control of Thermoacoustic Instabilities

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Active control of thermoacoustic instabilities has been an important topic of research for many years. In particular, adaptive-control approaches are continually investigated because of their potential ability to adjust to changing and unknown operating conditions. Most adaptive controllers are model based and require accurate knowledge of the plant to which they are applied; this is currently an extremely challenging solution for the limit-cycling combustion plant. A more versatile controller approach, based on a gradient descent algorithm, is presented here. The time-averaged gradient controller can automatically adapt to changing plants and can assume a variety of different controller architectures and actuator styles, without the need for any explicit knowledge of the plant dynamics. We present the controller in a variety of forms and provide simulation and experimental results illustrating its effectiveness in stabilizing an unstable Rijke tube combustor and a 50-kW kerosene-fueled lean direct-injection combustor.

Introduction

THERMOACOUSTIC instabilities have been an impediment to technology development since the 1950s. Most recently, attempts to reduce pollutant emissions from land-based gas turbines using lean fuel/air ratios have been hindered by this instability. The instability results from an interaction of the lightly damped acoustics of the combustion chamber and the heat release of the combustion process. Perturbations in the acoustic pressure and velocity cause the combustion process to produce a time-varying heat release. This in turn causes a time-varying expansion of gases that forces the acoustic system and results in a feedback loop. When the system becomes unstable, nonlinear effects prevent the oscillation amplitude from increasing without bound. The result is a limit-cycling condition that in many cases exhibits unacceptable amplitudes. Because of the wide range of operating conditions for gas turbines, passive solutions are hard to achieve, and active combustion control (ACC) is a promising alternative.

Many algorithmic approaches to active control of thermoacoustic instabilities have been investigated, and they are all hindered by significant limitations. Manually adjustable feedback controllers cannot effectively track changes in plant dynamics or combustor operating conditions. Model-based controller designs such as H-infinity and linear quadratic Gaussian require accurate knowledge of the plant to be controlled. Filtered-X least mean squares (LMS)-based adaptive controllers also require accurate plant models to prevent divergence of the weight adaptation process. In practice, modeling the unstable thermoacoustic plant through changing operating conditions has proven to be a very difficult task and is currently not practical for feedback controller design. Although online system identification is possible, it adds an additional level of complexity and uncertainty to the control system. Some gradient-based methods, such as extremum-seeking control and the triangular search algorithm, do not require system models, but are not as easily adapted to multiparameter controllers.

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In this paper, a simple and versatile algorithm for suppressing instabilities, hereafter referred to as the time-averaged-gradient (TAG) algorithm, is introduced. Generally, TAG control algorithms automatically determine the direction of the local gradient and adapt control parameters to minimize the mean-squared error of the input signal. Because the gradient is computed online, no a priori information regarding the detailed behavior of the thermoacoustic instability is required. The algorithm searches the performance surface, which is a function of the control parameters, and finds a solution that locally minimizes the mean-squared error. The algorithm is versatile enough to update any type of control parameters [finite impulse response (FIR) filter coefficients, gain and time-delay parameters, etc.] and operate with a wide variety of actuation schemes (proportional, on-off, subharmonic, etc.) with few or no algorithmic modifications.

The efficacy of such a simple control algorithm for the thermoacoustic instability problem is caused largely by the simple nature of this particular control problem. Although the thermoacoustic feedback loop is extraordinarily complicated with regard to the detailed physics, the control problem is dominated in practice by a single pair of acoustic poles crossing the imaginary axis caused by the gain of the heat-release subsystem. Stabilizing the system can usually be accomplished with extremely simple control structures. To maintain control over widely varying conditions, the controller must be made adaptive, and the general TAG scheme is ideally suited for modifications that allow it to be successfully used for ACC. It is emphasized that this class of gradient descent algorithm has been in existence previously; however, the application to ACC and the different parameterizations of the gradient variables is a contribution to the combustion control community.

The paper is organized as follows. A brief description of the control methods that have been applied to suppress thermoacoustic instabilities in the past is presented, noting some of the key advantages and disadvantages. Then, the TAG controller and update algorithms are presented along with their convergence properties. Simulations and experiments using the TAG controller are presented next to demonstrate its effectiveness for suppressing thermoacoustic instabilities, in general and in the face of nonstationary dynamics. Alternative implementations of the TAG controller and the appropriate choice of parameters that affect convergence are also discussed.

Background: Controller Designs for Suppression of Thermoacoustic Instabilities

A significant number of previous publications have discussed control algorithm designs that are able to suppress thermoacoustic

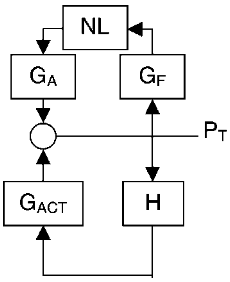

Fig. 1 Feedback control of self-excited plant.

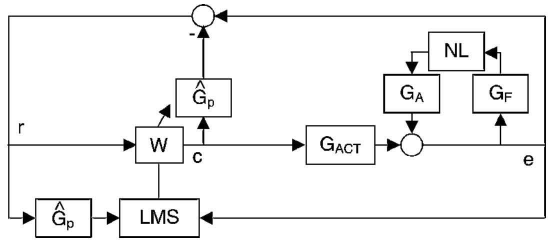

Fig. 2 Adaptive feedback, self-excited plant.

instabilities. The purpose of this section is to review prior work based on a specific category of such control algorithms, which can be labeled as adaptive signal-processing algorithms. Early research of adaptive combustion control tended towards adopting such algorithms because they were known to be easy to use and quite successful in suppressing both acoustic and vibration disturbances of many types. It was natural to try to extend those successes into the field of active combustion control. In fact, past investigations of adaptive signal-processing controllers has led to the TAG algorithm discussed in the following sections. Therefore, a brief description and history of those controller architectures will be helpful. First, however, we briefly review the ACC problem in general.

Figure 1 illustrates a simplified block diagram of a controlled, self-excited thermoacoustic system. The upper loop includes flame dynamics G_F, acoustic dynamics G_A, and a saturation-type nonlinearity. If the nonlinearity N_L were removed from the loop, one would observe that if the loop gain $|G_A G_F|$ of the acoustic and flame dynamics exceeds unity at a 360-deg phase crossing at least one closed-loop pole would be unstable. Using a describing function representation of the nonlinearity, one would calculate the existence of a self-excited oscillation (i.e., limit cycle) when $N_L G_A G_F = 1$. The oscillation of the limit cycle must be attenuated to acceptable levels to prevent damage to the combustor. A simple method for reducing the limit-cycle amplitude is to apply a fixed-gain feedback controller as shown in the lower loop of Fig. 1. The control filter H acts on the pressure signal measured from the combustor and generates a control pressure that forces the combustion environment through the actuator dynamics G_{ACT}. (For the Rijke tube experiments presented later, the actuator refers to an acoustic driver, whereas a proportional fuel injector is the actuator referred to for the kerosene LDI combustor experiments.) This type of feedback control has been widely investigated.

Fixed-gain feedback controllers, often implemented using a tunable phase delay, have been widely investigated in the control of combustion instabilities.[1–3] The primary impediment to practical application of fixed-gain controllers is their inability to adapt to varying operating conditions. As combustor operating conditions change, the flame dynamics are modified, and the acoustic temperature profile shifts, causing the limit cycle to change in frequency and amplitude. The typically narrowband performance of fixed-gain feedback controllers cannot accommodate the dynamic changes that can occur for widely varying operating conditions.

Feedforward control relies on the availability of a separate reference signal that is related to some disturbance input to a plant, but will be unaffected by the control input to the plant. Feedforward controllers are often implemented using adaptive filtering methods. The LMS or recursive least squares (RLS) algorithms have been extremely popular and were among the first type of adaptive control algorithms used for ACC. Although conventional feedforward disturbance suppression for open-loop stable plants has the advantage of guaranteed stability, it cannot alter the system poles. For linearly unstable, limit-cycling plants, such as a thermoacoustically unstable combustor, stabilizing control must be achieved by drawing the unstable poles back to the stable half-plane. Interestingly, the application of feedforward control for suppression of combustor instabilities always creates a feedback path so that the unstable poles are altered. This is discussed next.

Billoud et al.[4] and Kemal and Bowman[5] were among the first to examine adaptive methods for controlling combustion instabilities. Billoud's group was the first to attempt using known adaptive signal-processing techniques for ACC. Their work investigated the LMS algorithm. Figure 2 illustrates a typical implementation of a filtered-X LMS adaptive controller applied to the combustion plant. Upon close examination, it can be seen that the use of the LMS algorithm for combustion control leads to a feedback control architecture (unlike feedforward disturbance suppression for a stable plant), and it is thus capable of altering the system dynamics. For this application it is also noted that the plant estimate is required for both canceling the effects of using the error signal as the reference signal and ensuring that the gradient estimate is correct in the filtered-X portion of the algorithm. The primary difficulty encountered in implementing LMS-based control methods, for any application, is the need for a plant model. Because the thermoacoustic system is unstable, obtaining the required plant model was extremely challenging to previous ACC researchers.[6,7] Fortunately, a new technique for the plant identification (ID) process has been discussed in detail in Vaudrey,[8] and it is shown that this technique alleviates previous problems related to the ID process for LMS ACC control. However, even when the uncertainty inherent in the plant estimate can be resolved, there are other reasons why LMS-based feedback control might not be the most robust solution to the adaptive ACC problem. Two other issues, related to controller-induced feedback instabilities and gradient convergence problems, are described in Vaudrey et al.[9]

Therefore, the difficulties in practical implementation of filtered-X based control approaches have motivated the investigation of new adaptive algorithms for controlling thermoacoustic instabilities. Recent adaptive-control algorithms that have been discussed include extremum-seeking algorithms,[10] triangular-search algorithms,[11] and the time-averaged gradient algorithms[12] presented here. None of these methods require explicit knowledge of the plant dynamics in order to suppress the limit cycle. Extremum-seeking control, the triangular-search algorithm, and the time-averaged gradient algorithm presented here are all conceptually similar approaches to minimizing a specific cost function by continually updating system parameters to reach a local minimum. Each approach perturbs a control parameter by a specified amount and reevaluates the cost. A decision is then made to update or not update the control filter based on a measured improvement in performance. The time-averaged gradient control technique underlies the TAG algorithm and is discussed in detail next.

Time-Averaged Gradient Control

Gradient-descent algorithms, such as Newton's method and the method of steepest descent, are capable of searching a performance surface and locally minimizing multidimensional functions. Their primary limitation is that the higher-order filters necessary to control large bandwidths require longer adaptation times than their LMS-driven counterparts of lower order. Recalling that the thermoacoustic instability problem is very narrowband, we proceed with the discussion of these algorithms.

Gradient-descent algorithms measure the gradient of a cost function and continually update the independent parameters to move in the direction of the minimum cost. The method of steepest descent is a simple implementation of this concept and can be represented by the following weight update equation:

$$w_{n+1} = w_n - \mu \xi'(w_n) \tag{1}$$

where the convergence parameter μ controls the speed of the weight update. Equation (1) represents the update for a single weight at

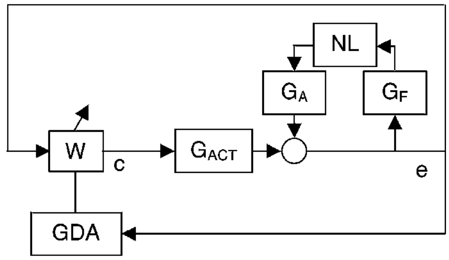

Fig. 3 TAG control of self-excited plant.

iteration n, but can be extended to the multiweight case by replacing the scalar parameters with vectors. [It is also clarified that the weights of Eq. (1) can form the coefficients of an adaptive filter or some other parameterized controller shown as W in Fig. 3.] The gradient of the cost function for a given update is computed using a measurement of the cost at the current weight location and a perturbed weight location by the formula

$$\xi'(w) = [\xi(w) - \xi(w - \delta)]/\delta \tag{2}$$

where the cost is given by the mean-squared error (MSE) of some signal e:

$$\xi(w) = \frac{1}{N}\sum_{n=0}^{N} e^2(n) \tag{3}$$

This method minimizes the cost as a function of each filter weight so that the computational complexity increases with the number of weights, resulting in a lengthy convergence time for large filter sizes. In the case of the thermoacoustic instability, it might be sufficient to adapt as few as two weight parameters to attain a stable operating point and cost minimization. $2N$ samples are collected before a single weight update can occur. For a two-weight adaptive filter $3N$ samples are collected after two separate perturbations in order to update the entire filter.

Newton's method is an alternative gradient-descent method that finds a zero of an arbitrary function. In the case of cost minimization, the minimum mean-squared error might never become zero. However, the gradient of the cost will become zero at the local minimum. Newton's method derives from the definition of the derivative and can be expressed as

$$x_{n+1} = x_n - f(x)/f'(x) \tag{4}$$

In general applications we are only concerned with the multiparameter case where the weight update becomes

$$W_{n+1} = W_n - [\xi''(x)]^{-1}\xi'(W) \tag{5}$$

and ξ'' is the matrix of second partial derivatives, referred to as the Hessian matrix. Computing and inverting this matrix greatly increases the computational complexity of the algorithm. The potential for singularity of the Hessian matrix also introduces complications.

Each of the weight-update algorithms has advantages and disadvantages. Computationally, the steepest descent is able to update the weights much faster than Newton's method. However, Newton's method can converge to the optimal solution faster than the steepest-descent algorithm. This is because the steepest-descent algorithm moves in the direction of the steepest local gradient, which is only in the direction of the optimal solution if the initial condition is on a principle axis of the error surface. But faster convergence, which implies larger parameter steps, can be a liability when dealing with the nonquadratic cost surface generated by the feedback nature of the ACC control architecture shown in Fig. 3. This fact, coupled with the increased computational complexity and potential singularity problems of Newton's method, makes the steepest-descent technique a better choice for the thermoacoustic instability control solution.

There are two limitations inherent to the TAG control structure shown in Fig. 3: 1) feedback loops can generate instabilities, and 2) the cost function can contain local minima. Although the gradient-descent algorithm might drive the weights in the direction of the negative gradient, there is no assurance that a global minimum will be reached. There is also no guarantee that the minimum which is found will result in a stabilized system. Even with these constraints, the gradient-descent algorithm has outperformed its filtered-X counterpart in comparable simulations where the control-to-error path G_p can change from trial to trial. The two limitations are fairly easily dealt with in practical implementations. If a parameter step results in the control feedback loop going unstable, this instability can be sensed, the parameter step undone, and a smaller step size chosen. To alleviate the local minimum problem, the highly scheduled nature of real-world controllers will allow control gains near the global minimum to be swapped in as operating points are changed. In addition, some amount of random probing can be incorporated in the algorithm when the current operating cost is viewed as too high. This random probing will eventually cause the controller to find a better parameter set and extract itself from a local minimum condition. This is demonstrated later for the liquid fuel combustion control experiments.

Widrow and Stearns[13] examine the stability constraints of the steepest descent algorithm in detail. The convergence parameter μ must be less than the inverse of the maximum eigenvalue of the input correlation matrix to ensure stable convergence. In a practical setting the maximum eigenvalue can be approximated by the tap input power $[W_0 W_1][W_0 W_1]^T$. The other parameters that will affect the convergence of the TAG controller are the perturbation size δ and the average duration N. After the weight vector has converged to the optimal solution, a piecewise-constant perturbation will ensure continual tracking of plant changes. Small perturbations might cause slow convergence to the optimal but will reduce the minimum MSE after convergence. Alternatively, relatively large values of the perturbation might provide more accurate estimates of the local gradient permitting faster convergence; this is at the expense of an increased minimum MSE after convergence. The appropriate average duration N is largely dependent on the noise present in the system. For nonnoisy systems low average durations can generate accurate estimates of the local MSE and thus accurate computation of the gradient. Longer average durations will lead to longer convergence times because the weight cannot be updated until the gradient is computed. More detailed observations on the effects of these parameters on convergence and performance of the control of a thermoacoustic instability are presented in conjunction with simulations and experiments described next.

Simulation

The thermoacoustic instability and TAG controller using the steepest-descent algorithm were simulated using MATLAB.® The acoustic portion of the combustion plant was modeled as a lightly damped single acoustic mode while the flame dynamics were modeled as a low-pass filter. The modeled acoustic and flame dynamics were

$$G_A(z) = \frac{-300000}{z^2 + 43.98z + 1.21e6} \qquad G_f(z) = \frac{1500}{z + 1257} \tag{6}$$

A saturation nonlinearity was approximated using a hyperbolic tangent function. Although this is a crude model from a physical standpoint, it is sufficient to illustrate the controller performance because the spectrum and rms pressure of the physical system is typically dominated by a single-frequency limit cycle and its harmonics. The gain for the self-excited loop was chosen such that the limit cycle was established after only one second.

The actuator dynamics G_{ACT} in Fig. 3 include all dynamics associated with A/D and D/A conversion (i.e., time delay), the dynamics of the actuator, and the acoustic pressure response caused by actuation. Two different actuator paths were chosen to illustrate different convergence behaviors of the TAG algorithm. Both contained the acoustic dynamics, although one was a high-gain, delayed version and the other used a lower gain without delays; both are shown here:

$$G_{\mathrm{ACT}}(z) = \frac{-0.11z - 0.11}{z^7 - 1.525z^6 + 0.973z^5}$$

$$G_{\mathrm{ACT}}(z) = \frac{-0.011z - 0.011}{z^2 - 1.525z + 0.9729} \tag{7}$$

For most experiments discussed here, the TAG controller was a two-weight FIR filter whose coefficients were updated using the steepest descent algorithm. Deviations from this are explicitly noted.

Figure 4 illustrates how the feedback loop stability varies with a two-weight feedback controller. The stable region (darker) is surrounded by a larger region, where at least one closed-loop pole is unstable. The TAG controller drives the weights to the stable region as long as it does not encounter a local minimum on the performance surface. In addition, it will adapt away from higher-amplitude limit cycles that might be present in the unstable region of Fig. 4, thereby preventing controller-induced instabilities.

Figure 5 illustrates a normalized segment of the performance surface of the system of Fig. 3 when the actuator path contains delays. This corresponds to the same system for which the stability map of Fig. 4 was generated. It is apparent that a local minimum does exist on this performance surface. It is therefore possible that a small perturbation size might result in the algorithm settling to a local minimum.

Simulations indicate that 0.0625 s worth of average time is sufficient to provide convergence for very high noise levels and a 175-Hz instability sampled at 1600 Hz. In fact, the TAG algorithm simulations shown in Fig. 6 illustrate that the level of noise in the error signal does not interfere with convergence and that suppression of the peak instability is achievable to the noise power level. A simulation was conducted to examine the effects of additive noise on the performance of the TAG algorithm designed to stabilize a limit-cycling system. Three additive noise conditions were examined where the noise level was increased first by 60 dB, and then by another 20 dB, effectively decreasing the signal-to-noise ratio, where the signal was the limit-cycle frequency. Because the goal of the controller in this scenario is to stabilize a nonlinear system (rather than attenuate a tone), a loose correlation between algorithm performance and SNR was noted. As background noise increases and obscures the signal,

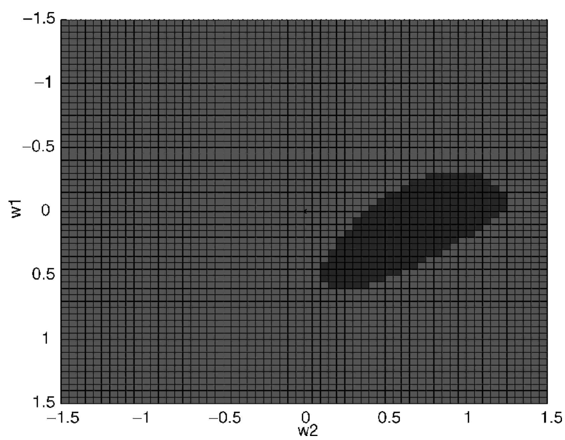

Fig. 4 High heat-release stability map.

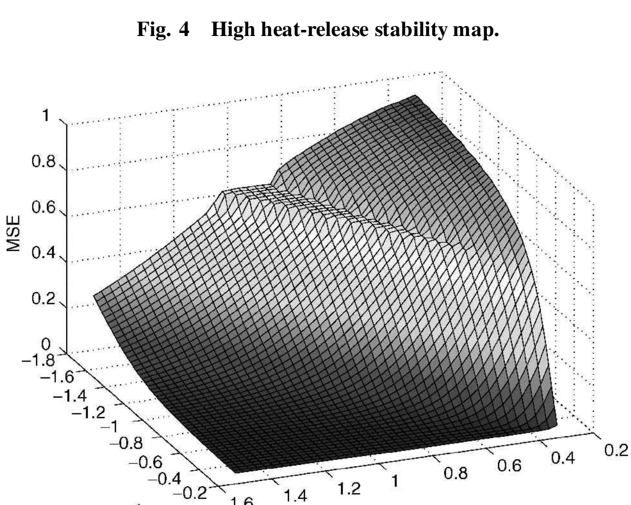

Fig. 5 Mean-squared-error performance surface.

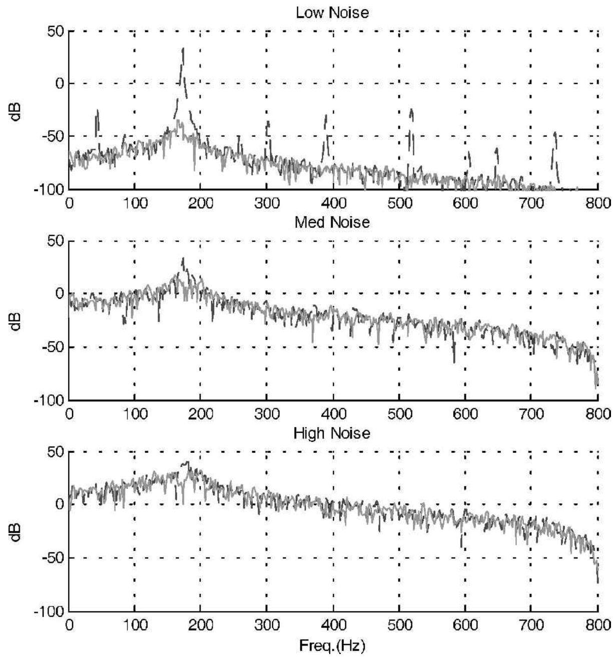

Fig. 6 Effect of varying noise level on TAG convergence.

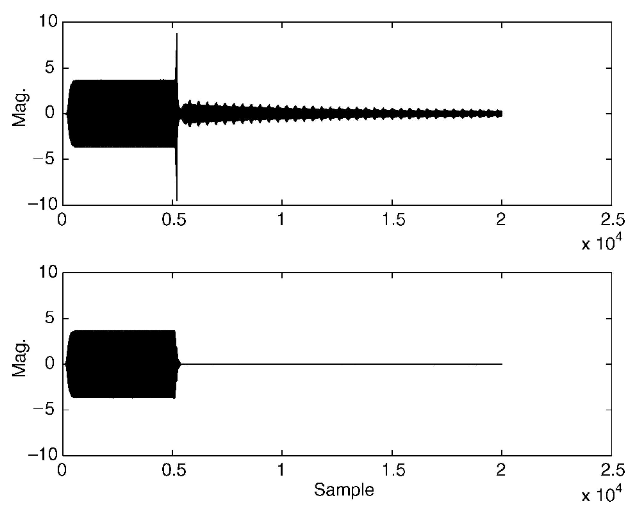

Fig. 7 Slow and fast TAG convergence.

the resulting control deteriorates. However, for each of the three cases examined in Fig. 6 the TAG controller design was able to stabilize the plant independent of the additive noise. Pathological cases could be created for these conditions where average times were too low and instability resulted. Nevertheless, the algorithm implemented in this system is relatively robust to additive noise.

Figure 7 illustrates the simulated TAG control of the limit cycle where the actuator dynamics include phase delay and added gain. For a nonoptimized choice of the perturbation size and convergence parameter (upper trace), the error signal is seen to initially increase in response to the perturbation. After the weights are corrected by the steepest-descent algorithm, the controller attenuates the limit cycle and begins driving it toward the minimum. The cycling amplitude seen in the error signal represents the weight perturbation and subsequent reevaluation of the gradient and weight update. The slow convergence and initial misdirection are corrected as seen in the lower trace, by changing the perturbation direction (adding a minus sign to δ) and increasing the magnitude of the convergence parameter μ.

As mentioned earlier, the convergence parameter size is limited by the stability of the algorithm. It is generally desirable to have a slightly overdamped response for noisy systems and gradient estimates to ensure continual convergence toward a minimum. Therefore, the size of the convergence parameter is governed by the input

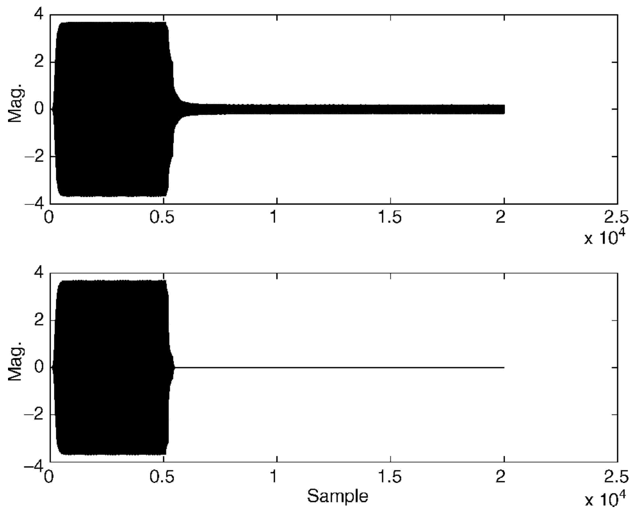

Fig. 8 Local minimum and optimized convergence.

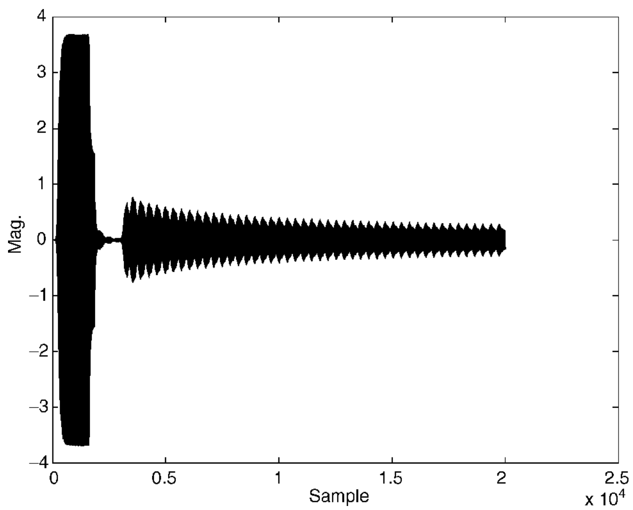

Fig. 9 Performance of TAG for changing plant dynamics.

power and the system noise and consequently controls the rate of convergence. The perturbation size is more system dependent. The amplitude of the perturbation must be sufficiently large so as to affect the rms level of the input signal in a measurable manner. If the perturbation is too small, the system will not be affected, no gradient will be computed, and the weights will not be updated. This is equivalent to finding a local minimum on the gradient surface, where sufficient weight excitation is not available to significantly alter the gradient estimation.

Figure 8 represents another simulation result where the actuator dynamics were a scaled version of the acoustic plant without additional delays. Here we see that for a nonoptimized choice of the convergence parameter and perturbation a new limit cycle (at a slightly lower frequency) is found. In this case significant adaptation stops, and the system has not been stabilized; a local minimum is found on the cost function. To remedy this (result shown in lower trace), the perturbation size was doubled, and the convergence parameter was increased. Larger perturbations in the weights permit more global gradient estimates, and the increased convergence parameter amplifies the gradient estimate to affect the weights more significantly. The result is a stabilized system.

Figure 9 illustrates a case where plant operating conditions were changed after the TAG controller achieved stabilizing performance for the first set of operating conditions. At sample number 3000 the heat-release portion of the plant was changed in magnitude and phase to represent a higher-gain operating condition. The TAG controller had not been previously optimized for this new operating condition, but was able to continue adapting, keeping the system stable, and significantly lower in amplitude than the original limit cycle shown from sample 500 to 1500. This simulation illustrates the TAG's ability to continually adapt to changing operating conditions.

The effects of the perturbation size and convergence parameter seen in simulation can be categorized into a set of rules that can adaptively control the convergence behavior of the TAG controller. These are summarized as follows:

1) If the MSE increases after the first perturbation, change the sign of the perturbation.

2) If the MSE increases after changing the sign, reduce the magnitude of the perturbation.

3) If the gradient is zero after the first several perturbations, increase the magnitude of the perturbation until a change in gradient is detected.

4) If the MSE is near the minimum MSE, decrease the magnitude of the perturbation to reduce the misadjustment noise.

5) If the system continually diverges, reduce the convergence parameter.

6) If the gradient is too small to change subsequent MSE calculations and the perturbation size has been increased, increase the convergence parameter.

Clearly these rules will interact with each other, and care must be taken when applying them in actual experimental settings so as to avoid local minima and divergence. A subset of these rules has been implemented in conjunction with the TAG controller in a quasi-experimental setting. The rule-based TAG was employed on a Texas Instruments (TI)-C31 digital signal processor (DSP) and used to control an analog electronics version of the single mode simulation just described. Implementation of the rules just described resulted in the desired effect of faster and more repeatable convergence. However, once a perturbation size was chosen, simply changing the direction of the perturbation when an increase in MSE was detected, resulted in stabilization for every trial. A maximum tap input power was determined based on the initial limit-cycle amplitude and the control actuator authority. This governed the choice of μ to guarantee stable convergence. This value was then reduced by approximately 10% to limit the speed of convergence.

Experimental Results

Two combustor test platforms at Virginia Polytechnic Institute and State University were used to examine the performance of the TAG controller in an experimental application. The first combustor examined is a closed-open tube combustor, nominally 1.5 m in length, burning premixed methane introduced at the closed-end of the tube. A pressure gauge at the closed end of the tube was used as the error sensor and was connected to a frequency analyzer, as well as the real-time DSP control system.

The second acoustic mode of the tube combustor is unstable when the burner is placed at the midlength of the tube. At low heat-release operating conditions, the frequency of the second mode is nominally 179 Hz, and the limit-cycle frequency is approximately the same. The control actuator was a 3-in. acoustic driver (speaker) coupled to the tube through a plenum and attached just above the flame. The actuator power was sufficient to achieve control at low heat-release conditions, corresponding to low equivalence ratios. At higher equivalence ratios, however, the increased heat release caused the self-excited loop gain to increase to the point that no controller could stabilize the system, although the limit-cycle amplitude could be reduced to some degree.

The Rijke tube combustor was set to operate with an equivalence ratio of 0.54 and a total volume flow of 130 cc/s. The two-weight TAG controller using the steepest-descent algorithm was then employed using a TI-C31 DSP control system. The uncontrolled input level of the pressure signal measurement was adjusted external to the DSP. Based on the input signal level, the convergence parameter was chosen as 0.4 to ensure gradient convergence. The perturbation value of 0.4 was found to be small enough to prevent excessive noise after convergence, yet large enough to provide a measurable change in the gradient. Typically this can be determined experimentally, or algorithmically, as described earlier. Figure 10 illustrates

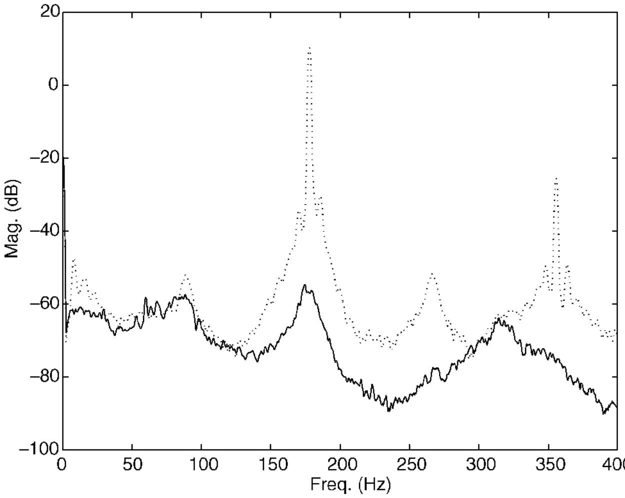

Fig. 10 Low heat-release tube combustor, TAG control result.

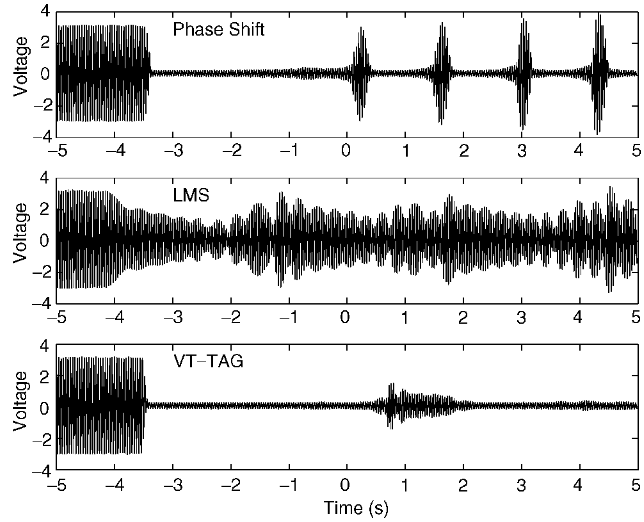

Fig. 11 High heat-release tube combustor, convergence comparison.

the converged TAG-controlled power spectrum of the pressure signal (solid) compared to the uncontrolled pressure spectrum (dotted). The TAG controller converged to a stabilizing solution within approximately 0.25 s and maintained control of the low-heat-release case indefinitely. The manually adjusted phase-shift controller required a magnitude of 0.519 and phase shift of 97 deg for best suppression, while the TAG algorithm converged to a magnitude of 0.84 and phase shift of 112 deg. Of course, it is not required for the TAG to exactly duplicate the phase shifter results but simply to get the weight values that lead to similar magnitude and phase values.

The TAG controller is a versatile method for controlling the combustion instability and can be applied in a variety of formats and parameterizations. Several different algorithm variants were investigated experimentally including the multiweight TAG (FIR filter using four weights) and the direct adaptation of gain and phase. The infinite impulse response (IIR) all-pass filter form of the gain-and-phase controller in the z domain is

$$H(z) = b\frac{z^{-2} + 2az^{-1} + a^2}{1 + 2az^{-1} + a^2z^{-2}} \tag{8}$$

This represents two first-order, all-pass filters in series where the control parameter a controls the phase delay and b controls the gain. Two all-pass filter sections permit 360 deg of phase adjustment at 175 Hz (the instability frequency) when sampled at 1600 Hz. If a is limited to remain between $+1$ and -1, the controller $H(z)$ is guaranteed to be stable while allowing controller poles to adapt. The gain and phase of the controller are perturbed and updated directly without altering the shape of the control filter's frequency response.

Both methods stabilized the Rijke tube combustor after brief transients, resulting in performances nearly identical to Fig. 10. The steepest-descent formulation of the weight update equation in no way limits the number of weights that can be updated or the form of the control filter. In the case of the Rijke tube combustor, two control weights were found to be sufficient to achieve stabilizing control. A notable benefit for additional weights is that they can more readily control the frequency response, preventing control-loop instabilities induced by the feedback controller. There is no specific advantage of choosing the gain-and-phase parameterization of Eq. (8) for the control of the Rijke tube combustor, as the performance was identical to that of the FIR filter control update. However, there might be instances where it is more advantageous to update controller parameters other than FIR filter coefficients; the TAG formulation can easily accommodate such changes.

To illustrate the versatility of the algorithm, it was also used to adapt a subharmonic controller trying to achieve stabilization of the Rijke tube. The output of a TAG-controlled two-weight FIR filter was frequency divided to produce an output with half the frequency of the input and a duty cycle of 50%. The second harmonic of the subharmonic control signal serves to attenuate the limit cycle in a linear system sense while the cycle stress on the actuator is reduced. The TAG controller requires no a priori information of this change because its only concern is to minimize the rms pressure of the error signal. The subharmonic application of the TAG algorithm led to limit-cycle amplitude reductions that were identical to those shown for the standard TAG implementation in Fig. 10.

Higher equivalence ratios, corresponding to higher self-excited loop gains, were not stabilizable using the low-power acoustic actuator employed in these experiments. However, it is informative to examine the performance of the TAG controller when compared to other controllers for this operating condition. Figure 11 illustrates three controlled time responses at the high heat-release condition. The upper trace represents a manually tuned phase-delay feedback controller. The intermittent behavior is a result of the actuator power limitations and changing plant dynamics in response to the control Vaudrey.[8] The second time response represents an adaptive feedback controller of the type shown in Fig. 2, employed at the same operating conditions. In both the fixed-gain and adaptive controllers, the intermittency amplitude was approximately the same as the original limit-cycle amplitude. However, when the TAG controller was used at these operating conditions the intermittency period increased, and the amplitude decreased significantly. In this case the adaptive TAG controller represents a more robust adaptive-control solution, even in the presence of actuator authority limitations.

The TAG controller was also evaluated using a 50-kW kerosene fuel combustor designed to exhibit a low-frequency instability, at approximately 100 Hz. The combustor operates with a fuel flow rate up to 1.5 gph. Details about the higher-power combustor are omitted here for brevity, but interested readers can refer to an investigation by DeCastro.[14] One primary concern about testing the TAG controller on a higher-power combustor is the possibility that the performance surface will not be unimodal, thereby leading to the possibility of converging at a local minimum. Recall that a probing function would be sufficient to avoid this potential problem.

Figure 12 illustrates the performance surface, that is, the instability peak amplitude vs operating condition, for the kerosene combustor. This is sufficient information to determine how the TAG algorithm will traverse the performance surface from any initial starting location. Figure 13 illustrates the results of a MATLAB simulation where the two weights of the TAG controller are allowed to adapt in accordance with the algorithm. The algorithm parameters for this case were $\mu = 0.4607$, $\delta = 0.38$, and $N = 998$ samples. Notice that the weights do encounter a local mininum that can stall the algorithm, at a phase value of approximately 150 deg. However, the addition of a simple piecewise-constant probing function was added to ensure that the algorithm constantly monitors the surrounding surface in the event that local minima are met. The probing

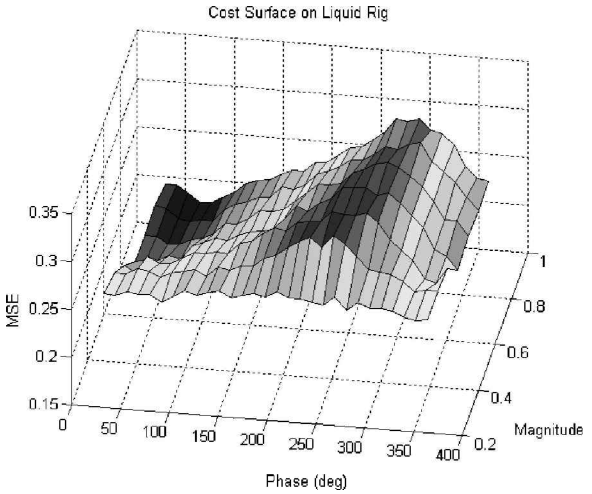

Fig. 12 Performance surface measured for 50-kW kerosene combustor rig.

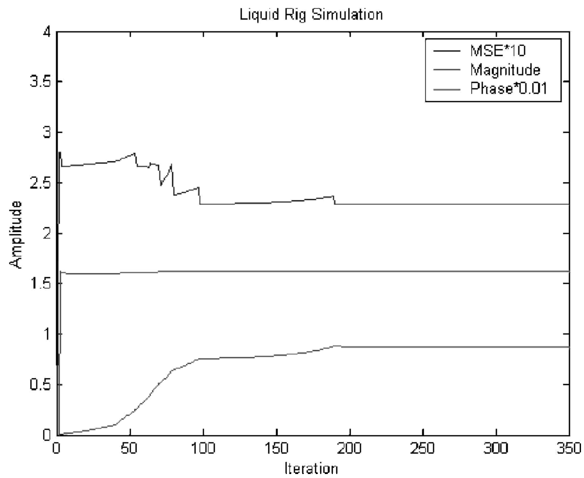

Fig. 13 Simulated TAG control using performance surface data for 50-kW combustor.

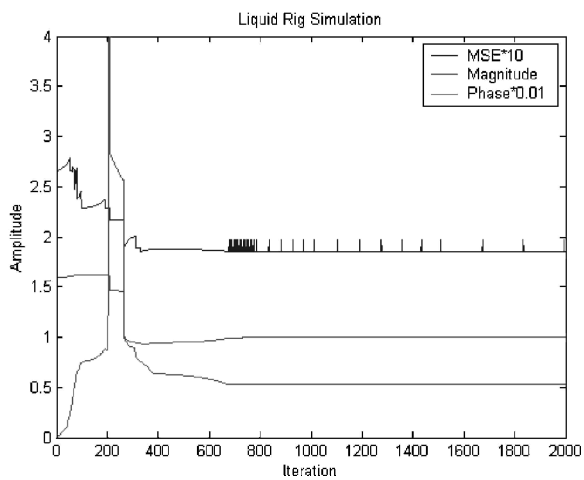

Fig. 14 Simulated TAG control using performance surface data for 50-kW combustor, with probing function added.

function simply explores the weight space once the weights seem to have converged, in order to ensure best performance. Figure 14 shows that the probing function successfully modifies the weights to reach the same minimum that would be found via a manual phase shifter (approximately 50–60 deg of phase from controller). This work is currently being extended; however, it was included here to emphasize that the TAG algorithm has merit for more realistic combustors than the tube combustor investigated earlier.

Conclusions

The time-averaged-gradient controller presented here was shown to be an effective and versatile method for controlling thermoacoustic instabilities. Requiring no plant estimate or model, the steepest-descent algorithm effectively moves a wide variety of control parameters toward a solution that consistently minimizes the mean-squared error of the input signal.

Effective control of a Rijke tube combustor was achieved experimentally when the TAG controller was a two-weight FIR filter, an all-pass IIR filter, a four-weight FIR filter, and a subharmonic frequency division of the output of a two-weight FIR filter. Control simulations for a 50-kW combustor, based on actual operating data, also indicated the success of the TAG algorithm that was modified to include a probing function.

Because of its ease of implementation, control effectiveness, and versatility, it holds advantages over other controllers we have discussed. Fast and repeatable convergence of the TAG controller is dependent upon several algorithmic parameters. Certain observations, simulations, and experimental results presented here help define the effects of these parameters on the overall performance. Initial investigation into automatic selection of these parameters was presented and shown to be effective. Future developments will include continued investigation of automatically selecting the convergence parameter, averaging-interval, and perturbation of the TAG controller in order to make it more system independent.

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